ARTICLES

THE NEAR CERTAINTY OF PATENT ASSERTION ENTITY VICTORY IN PORTFOLIO PATENT LITIGATION

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I. INTRODUCTION

A patent assertion entity (PAE) is a company that is organized for the sole purpose of acquiring and enforcing patents.[1] Similar to other non-practicing entities (NPE), a PAE does not practice the claims set forth in its patents.[2] A PAE not only does not manufacture, distribute, or sell any products or services, it, unlike many other NPEs such as universities and sole inventors, has no direct or indirect interest in facilitating the sale of products or services by others.[3] Because PAEs are not viewed as being inventive or productive, their social utility has come into question.[4] This is especially so because of a great increase in the number of patent suits filed by PAEs.[5] Recently, the number of patent suits filed by PAEs has begun to exceed the number of patent suits filed by all other entities.[6] Moreover, the PAE business model appears to be increasingly successful; a recent article estimates that defendants and licensees paid PAEs $29 billion in 2011 for their patent rights, a four-fold increase from the figure for 2005.[7]

Given the success of the PAE business model, one may think that PAEs are especially good in identifying, acquiring, and asserting high-value patents. This Article argues otherwise. Based on the insight and

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1. A recent White House report defined a PAE as a firm that

> do[es] not practice the patents [it owns] and instead [engages] in aggressive litigation to collect license and other fees from alleged infringers. A review of the evidence suggests that on balance, such patent assertion entities (PAEs) (also known as "patent trolls") have had a negative impact on innovation and economic growth.

EXECUTIVE OFFICE OF THE PRESIDENT, PATENT ASSERTION AND U.S. INNOVATION 2 (2013), *available at* http://www.whitehouse.gov/sites/default/files/docs/patent_report.pdf.

2. *Id.* at 4.

3. *Id.* at 3–4.

4. *Id.* at 2. However, more broadly, NPEs are recognized as having social utility because they serve a useful patent intermediary function in facilitating the transfer of technology from innovators to the entities that are in the best position to exploit innovations. *Id*. at 2–3.

5. *Id.* at 5.

6. *Id.* at 4 (noting that PAEs accounted for 62% of all patent suits brought in 2012).

7. James E. Bessen & Michael J. Meurer, *The Direct Costs From NPE Disputes* 19, 24, 31 (Boston Univ. Sch. Law, Working Paper No. 12-34, 2012), *available at* http://www.bu.edu/law/faculty/scholarship/workingpapers/2012.html.

discussion of a 2005 article by Gideon Parchomovsky,[8] the observation that PAEs most often assert multiple patents (that is, a portfolio of patents) against a product or a line of products, and considerations of probability,[9] this Article argues that PAEs are virtually assured of victory. For that reason, their success is not a direct reflection of the ability of PAEs to identify high-value patents.

The main contribution of this Article is articulating the mathematical basis for the observation that a PAE that asserts a portfolio of patents against a product or a product line is assured of success; that is, a finding of patent liability on the part of the accused infringer in connection with at least one asserted patent. This liability is assured where the set of material issues in dispute for the asserted portfolio of patents are mathematically independent and a large number of patents are asserted. The mathematical independence of a set of material issues means that the resolution of any one material issue does not affect the probabilities of resolution of the others. The mathematical independence assumption should hold where the patents selected for assertion against the product or line of products are from different technology areas.[10] For example, this assumption likely holds when the patents asserted against a successful tablet computer include patents directed to processor instruction execution, processing for rendering graphics display images, and the construction of the liquid crystal display of the tablet computer.[11] Each of the material legal issues that will be contested by the parties in a patent litigation involving such patents will likely be different, justifying the assumption of mathematically independent probabilities for the patentee's prevailing on these issues.[12]

Moreover, given the current state of patent litigation in the United States, the process of determination of whether each patent is infringed will usually involve the resolution of 5–10 material legal issues. A PAE,

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8. *See generally* Gideon Parchomovsky & K. Polk Wagner, *Patent Portfolios*, 154 U. PA. L. REV. 1, 60–65 (2005).

9. These probability considerations are discussed in *infra* Part IV.

10. This is because material issues relating to a patent in one technology area should not relate to material issues relating to a patent in another technology area. PAEs frequently assert patents in different technology areas against a single product or product line of a hi-tech consumer goods company. *See, e.g.*, Certain Consumer Electronics and Display Devices and Products Containing Same, Inv. No. 337-TA-836, *available at* http://www.itcblog.com/wp-content/uploads/2013/05/gpcomplaint.pdf [hereinafter Certain Consumer Electronics Complaint]. The case eventually settled before an ITC decision. Additionally, the material issues in dispute in connection with a specific patent can usually be reduced to a core set of independent material issues.

11. *See, e.g.*, *id.*

12. *But see* Parchomovsky & Wagner, *supra* note 8, at 33 n.120 (asserting that the assumption of absolute mathematical independence is too strong, but that the likelihood of success of an infringement suit still increases as portfolio size increases).

before it files suit, is usually able to identify and acquire those patents that present the most favorable set of probabilities for the material issues that need to be resolved for determination of patent infringement liability. However, regardless of the exact values of these probabilities, the PAE may, at least theoretically, assure itself of victory by asserting a sufficiently large number of patents against the target product or product line. In fact, this Article shows that in the limit of assertion of a sufficiently large number of patents, the outcome is insensitive to the exact values of probabilities of prevailing on the material legal issues.

Because of the virtual certainty of PAE victory in portfolio patent litigation, this Article predicts that many reforms that have been implemented or that are currently being discussed will likely have small or negligible impact in eliminating or materially reducing the number of PAE litigations. For example, while shifting the cost of litigation onto the losing party may increase the perceived risk of a PAE filing a patent suit, the fact that the PAE is assured of victory by asserting a sufficient number of patents would likely lead it to nevertheless file the suit.

A corollary of the result that a PAE is assured of victory by asserting a sufficient number of patents is that the value of a specific patent likely has strong dependence on how it is asserted—for example, on whether it is asserted by itself or in conjunction with other patents, and, if the latter, whether the different patents are asserted against the same or different products and product lines. Known methodologies for valuing patents, to the extent they do not consider how the value proposition may be impacted by assertion together with other patents, may be deficient.

Part II of this Article reviews the generally unsatisfactory efforts up to now in valuing patents. Part III discusses the Parchomovsky article, which provided the insight that "the true value of patents lies not in their individual worth, but in their aggregation into a collection of related patents—a patent portfolio."[13] Part IV discusses the mathematical basis for the assertion that the patentee's probability of prevailing on at least one asserted patent increases as the number of asserted patents in the portfolio increases. This discussion is based on, among other things, consideration of the number of material legal issues that are observed in modern patent infringement litigation. However, the analysis is not sensitive to the exact values of these probabilities. Part V applies the ideas that are discussed in Part IV to the specific case of PAE patent litigation. Part VI discusses results of the analysis, evaluates various policy options directed to PAEs in the context of the findings of this Article, and provides a few conclusions.

13. *Id.* at 77.

II. THE PROBLEM OF VALUATION OF PATENTS

The valuation of patents is an unsolved problem.[14] Much has been written about the valuation of patents, but there is no universally accepted, precise methodology for valuing patents.[15] This is becoming an increasingly serious problem, because the United States and world economies are increasingly based on a foundation of technological property, with an ever-greater number of transactions and disputes relating to patents arising among companies. The lack of a widely accepted methodology to value patents likely leads to great legal uncertainty and hinders the efficient transfer and utilization of technology.

Companies, patent professionals, practitioners, and judges, to complete transactions involving the transfer of technology or resolve patent disputes, necessarily resort to various methodologies in directly or indirectly valuing patents.[16] But often, the results of these methodologies appear to be at least partially random, and in some cases, measurably wrong.[17]

In a recent example of how even the most sophisticated patent valuation professionals may be way off in their valuations, the patent valuation firm M-Cam, Inc. had appraised AOL's patent portfolio at $290 million, with most other experts valuing the portfolio as being worth no more than $300 million.[18] Microsoft purchased the AOL portfolio at a price that was more than $1 billion.[19] In another recent example, Kodak, facing bankruptcy, desired to sell its patent portfolio and expected a sum exceeding $2.5 billion.[20] Kodak was able to sell its portfolio for only $525 million.[21]

Uncertainty regarding valuation also pervades patent infringement damages decisions in the judicial system. For example, for approximately fifteen years, one method of calculating reasonable-royalty compensatory damages upon a district court's finding of patent

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14. *Id.* at 4.

15. *Id.* at 4 n.1.

16. *See id.* at 19–26 (discussing different theories of patent valuation).

17. *See id.*

18. Bruce Berman, *Could AOL Patent Sale Have Netted More than $1B?*, IP CLOSEUP (Apr. 10, 2012), http://ipcloseup.wordpress.com/2012/04/10/could-aol-patent-sale-have-netted-more-than-1b/.

19. *Id.*

20. Andrew Martin, *Kodak to Sell Digital Imaging Patents for $525 Million*, N.Y. TIMES, Dec. 19, 2012, http://www.nytimes.com/2012/12/20/business/kodak-to-sell-patents-for-525-million.html.

21. These are summarized in *infra* Part II.A–F which also discusses the appropriateness of applying the methodologies to value patents.

infringement involved the so-called "25% rule."[22] This rule had been used as a "tool [for approximating] the reasonable royalty rate that the manufacturer of a patented product would be willing to offer to pay the patentee during a hypothetical negotiation."[23] According to this rule, 25% of the infringer's profits from sale or use of infringing products are designated as a baseline royalty, which can be increased or decreased based on the Georgia-Pacific factors.[24] The Federal Circuit in 2011 rejected the 25% rule, finding that it was a "fundamentally flawed tool for determining a baseline royalty in a hypothetical negotiation,"[25] and also finding that evidence relating to the rule was "inadmissible under Daubert and the Federal Rules of Evidence."[26] In so finding, the appeals court vacated a jury damages award of $388 million.[27]

The damages awards for patent cases not involving the 25% rule also indicate that there is great uncertainty regarding valuation of patents.[28] For example, most of the top ten patent infringement damages awards of 2012 were vacated, remanded or reduced on appeal.[29] Additionally, the difference between median awards of juries and of the bench has increased immensely over time with the median jury award during the period 2007—2012 being 40 times as large as the median bench award, compared to a discrepancy of only a factor of 1.2 during the period 1995—2000.[30] In relatively recent examples that are dramatic, a $357 million jury award in litigation involving Lucent and Microsoft was reduced to approximately $26 million,[31] and a $1 billion jury award in litigation involving Apple and Samsung was reduced to approximately $599 million.[32]

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22. *See, e.g.*, Fonar Corp. v. Gen. Elec. Co., 107 F.3d 1543, 1553 (Fed. Cir. 1997) (noting that Fonar's expert witness "testified that one-quarter to one-third of the anticipated profits on the sale of the infringing machines would have constituted a reasonable royalty").

23. Uniloc USA, Inc. v. Microsoft Corp., 632 F.3d 1292, 1312 (Fed. Cir. 2011).

24. Georgia-Pacific Corp. v. U.S. Plywood Corp. set forth fifteen factors that may be considered under the most commonly used method to determine reasonable royalty damages in a patent case. *See* Ga.-Pac. Corp. v. U.S. Plywood Corp., 318 F. Supp. 1116, 1120 (S.D.N.Y. 1970), *mod. and aff'd*, 446 F.2d 295 (2d Cir. 1971), *cert. denied*, 404 U.S. 870 (1971).

25. *Uniloc USA*, 632 F.3d at 1315.

26. *Id.*

27. *Id*. at 1311, 1323.

28. *See* PRICEWATERHOUSECOOPERS, PATENT LITIGATION STUDY 8 (2013), *available at* http://www.pwc.com/us/en/forensic-services/publications/2013-patent-litigation-study.jhtml (noting that few high-value patent damages awards have been upheld).

29. *Id*.

30. *Id*. at 10.

31. *See* Lucent Techs., Inc. v. Gateway, Inc., 580 F.3d 1301, 1309 (Fed. Cir. 2009); *see also* Lucent Techs., Inc. v. Microsoft Corp., 837 F. Supp. 2d 1107, 1110 (S.D. Cal. 2011).

32. *See* Apple, Inc. v. Samsung Elecs. Co., 926 F. Supp. 2d 1100, 1103, 1120 (N.D. Cal. 2013).

Valuation uncertainty also has an impact during prosecution.[33] Often, a company cannot know with any degree of certainty the commercial benefits that a particular invention, if patented, would yield.[34] Given this uncertainty, companies often decide to expend the same share of resources on each application, despite the almost certain knowledge that some of the inventions, when patented, will yield commercial benefits significantly larger than the commercial benefits yielded by other patents in the portfolio.[35]

Patents may be hard to value because their valuation involves more uncertainty and complexity compared to traditional asset classes that are more easily understood and which are often used as bases of comparison to the patent right.[36] For example, patents are sometimes compared to stock or options on stock.[37] Although predicting stock prices is also fraught with difficulties, there are well-understood methodologies for predicting the value of options on stocks.[38] In any case, stock markets provide a market valuation for publicly traded stock nearly continuously. There is no analogous market that provides a value for patents at frequent time intervals.[39] Furthermore, stock prices may be estimated from the estimated total value of the relevant company, because stock represents a predetermined percentage of ownership of the company to which it pertains. It is not as easy to correlate the value of one or more patents owned by a company to the company's net worth, or even the entire value of its intellectual property portfolio.

Given that the patent right is a property right, patents may also be

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33. *See* Robert Pitkethly, *The Valuation of Patents: A Review of Patent Valuation Methods With Consideration of Option Based Methods and the Potential for Further Research* 1, 3 (Univ. of Cambridge Judge Inst. of Mgmt. Studies, Working Paper No. 21/97, 1997) (discussing the importance of realistic patent damages valuations to overcome judicial allegations of overly speculative assertions).

34. *See id.* at 3–5; *see also* Ted Sichelman, *Commercializing Patents*, 62 STAN. L. REV. 341, 343 (2010) (arguing that patent law contributes to the lack of commercialization of patents).

35. *Id.*

36. *See* WILLIAM J. MURPHY ET AL., PATENT VALUATION: IMPROVING DECISION MAKING THROUGH ANALYSIS 8, 121–22, 127, 129 (2012).

37. F. Russel Denton & Paul J. Heald, *Random Walks, Non-Cooperative Games, and the Complex Mathematics of Patent Pricing*, 55 RUTGERS L. REV. 1175, 1194–96 (2003).

38. These methodologies are summarized in Part II of this Article, which also discusses the appropriateness of applying the methodologies to value patents.

39. *See* WILLIAM J. MURPHY ET AL., PATENT VALUATION: IMPROVING DECISION MAKING THROUGH ANALYSIS 201–02 (2012). Patent auctions have recently begun to be held from time to time. *Id.* Up until now, they have not been organized regularly. *Id.* Furthermore, in these auctions, patents are usually offered in groups rather than on an individual basis. *Id.* Although patent auctions represent an improvement to the patent system in connection with the valuation of patents, they are nowhere near to providing the continuous and reliable information regarding valuation that the stock market provides in connection with the valuation of stock. *Id.*

compared to real property.[40] For example, the right to exclude provided by a patent has been compared to the right of a landowner to exclude third parties from the owned land.[41] However, once again, there are too many differences for methodologies for valuation of real property to usefully apply to valuation of patents. Similar to the situation with stocks and options, there usually is an active market for real property that values real property at relatively frequent time intervals. For example, real estate agents and the listings they keep and advertise serve such a function.[42] Additionally, publicly accessible property records usually contain information on the sales prices of parcels of land. An active market does not exist for patents.[43] Another important difference is that patents may be put to more complex uses compared to real property. For example, a unit of real property may usually only be rented to one person or entity at a time. However, a patent may be licensed to more than one person at a time, and may be so licensed for the same use or a different use. Additionally, a patent may be packaged with other patents and property and licensed together in one or more pools.[44] There are usually no such analogous uses of real property. The increased complexity arising from the possibility to license to multiple parties, finely carve out particular uses of the patented invention, and for bundling together rights under other patents and intellectual property likely also renders valuation of patents more difficult than doing so for real property.[45]

A. *Methodologies for Valuing Patents*

Despite the difficulties in valuing a patent, it is often necessary to assign a value in commercial and legal contexts.[46] The parties to a

40. *See* Frank H. Easterbrook, *Intellectual Property is Still Property*, 13 HARV. J.L. & PUB. POL'Y 108, 109 (1990) (likening patent rights to real property rights).

41. *See, e.g.*, Adam Mossoff, *Exclusion and Exclusive Use in Patent Law*, 22 HARV. J.L. & TECH. 321, 323, 347–49 (2009) (noting the shared right to exclude in patents and property); *see also* Mark A. Lemley, *Ignoring Patents*, 2008 MICH. ST. L. REV. 19, 23–25, 23 n.22 (2008) (discussing the analogy between patents and real property and citing to other discussions of this analogy).

42. *See* Malcolm T. Meeks & Charles A. Eldering, *Patent Valuation: Aren't We Forgetting Something? Making the Case for Claims Analysis in Patent Valuation by Proposing a Patent Valuation Method and a Patent-Specific Discount Rate Using the CAPM*, 9 NW. J. TECH. & INTELL. PROP. 194, 202 (2010).

43. *See* MURPHY ET AL., *supra* note 36, at 201–02.

44. *See* Joel I. Klein, Acting Assistant Attorney Gen., U.S. Dep't of Justice, Cross-Licensing and Antitrust Law, Address Before the American Intellectual Property Law Association 3 n.3 (May 2, 1997), *available at* http://www.justice.gov/atr/public/speeches/ 1118.pdf.

45. That is, there are a greater number of states or possibilities to consider in transactions involving a patent compared to a piece of real property.

46. *See* Robert Pitkethly, *The Valuation of Patents: A Review of Patent Valuation*

licensing or assignment transaction must effectively do so when agreeing to an applicable royalty for the license or a sale price for the assignment.[47] A judge must also do so when determining damages in a patent infringement suit, as discussed in Part II of this Article.

B. *Discounted Cash Flow and Income-Based Methods*

In cases in which it is possible to estimate future cash flows that arise from the patent, it may be possible to calculate the net present value of such cash flows as the estimate of the patent's value.[48] Doing so requires discounting the cash flows by the risk-free interest rate to account for the time value of money, and possibly by other factors to account for uncertainty regarding whether the projected cash flows will actually materialize.[49]

It is not possible, in many cases, to estimate the future cash flow that a patent will provide, especially if the patent is new or has not yet been exploited.[50] For that reason, use of the discounted cash flow, or DCF, method is generally limited to very specific circumstances. Even in cases in which future cash flows may be estimated with some accuracy, the DCF method may fail to provide for non-cash flow related value, such as the defensive, deterrent value a patent provides against competitors.[51]

C. *Market-Based Methods*

In a market-based valuation, a patent is valued based on the known or well-estimated value of another comparable patent.[52] Such a method may be sufficiently accurate where the comparable patent was acquired or sold in a transaction comparable to the one involving the patent that is being valued.[53] However, if the comparable patent and the patent to be valued are to be used differently in the respective post-transaction periods, then the market-based method may not provide an accurate value.[54]

Methods With Consideration of Option Based Methods and the Potential for Further Research 1 (Univ. of Cambridge Judge Inst. of Mgmt. Studies, Working Paper No. 21/97, 1997).

47. *Id.* at 3.

48. *See* MURPHY ET AL., *supra* note 36, at 121–31 (providing examples and calculations for discounted cash flow analyses as applied to patents).

49. *Id.*

50. *See* Robert Pitkethly, *The Valuation of Patents: A Review of Patent Valuation Methods With Consideration of Option Based Methods and the Potential for Further Research* 1, 8–9 (Univ. of Cambridge Judge Inst. of Mgmt. Studies, Working Paper No. 21/97, 1997).

51. *See id.* at 9.

52. *See id.* at 7.

53 *See id.* at 8–9.

54. *See* Liina Tonisson & Dr. Lutz Maicher, *Patents, Their Importance and Valuation*

One subclass of market-based methods for valuation involves calculation of the residual value after deducting all other known asset values from the market value of a company whose value is known.[55] However, the residual value will often correspond to the total value of a large number of intangible assets so that the value of a specific intangible asset from among that set is not easily obtainable.[56]

D. *Cost-Based Methods*

In perhaps the methodology that yields the least accurate and least precise valuations, one can estimate a value for a patent based on the historical cost of obtaining the patent, or the current cost of trying to obtain a similar patent.[57] Such methods do not account for or value the future benefits the patent may provide, and, for that reason, do not usually provide useful valuations.[58] Such methods are frequently used, however, in historical cost-based accounting systems or for purposes of calculating taxes.[59]

E. *Indicator-Based Methods*

Indicator-based methods can determine the relative value of a patent in comparison to a set of other patents. A relative value that is so determined may then be used to adjust a valuation that is determined in accordance with one of the other methodologies to yield a better estimate of the value of the patent.

One early example of such an indicator-based method references renewal data. The decision to pay a renewal fee to maintain a patent in force indicates a judgment on the part of the patent holder that the value of the patent is in excess of the renewal fee.[60] Thus, tracking renewal data for a patent provides information regarding the value of the patent. More recently developed examples include using one or more of the number of forward citations in other patents of the patent being valued, the number of backward citations in the patent being valued to other

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Methods 10 (Fraunhofer MOEZ Working Paper, Issue No. 3/2012, 2012), *available at* http://www.moez.fraunhofer.de/content/dam/moez/de/documents/Working_Paper/WP2012%20Nr.1_Patent-Valua tion_final.pdf.

55. *See* Robert Pitkethly, *The Valuation of Patents: A Review of Patent Valuation Methods With Consideration of Option Based Methods and the Potential for Further Research* 1, 7 (Univ. of Cambridge Judge Inst. of Mgmt. Studies, Working Paper No. 21/97, 1997) (citing Russell L. Parr, *Fair Rates of Return*, PATENT WORLD 36–41 (July 1988)).

56. *See id.* at 7.

57. *See id.* at 6.

58. *Id.* at 16.

59. *See id.* at 6.

60. *See id.* at 17–18.

patents and the age of those patents, market value of the company that owns the patent being valued, and the number of oppositions or other post-issuance proceedings filed in connection with the patent.[61]

F. *Option-Based Methods*

There have been attempts to value patents based on the theory of valuation of financial options. Financial options, such as call or put options on shares in a company may be valued based on the so-called Black-Scholes analysis.[62] In the simplest form of the analysis involving the value of a European call option, share prices are assumed to follow a lognormal random walk, the risk-free interest rate and the volatility of the shares are considered to be constant, the shares are assumed to pay no dividends, there are no transaction costs, there are no arbitrage possibilities, trading of the shares takes place continuously, short selling is permissible, and the shares are divisible.[63] Under such conditions, a deterministic (non-stochastic) partial differential equation for the option value can be obtained and solved.[64] The solution expresses the option value as a function of current asset price and time, with the exercise price, risk-free interest rate and the volatility of the shares set forth as constant parameters.[65] In the standard derivation of the solution, a crucial step involves construction of a portfolio that includes the call option and a number of shares in a way that eliminates a stochastic factor that is present in the equation due to the assumption that the share prices follow a lognormal random walk.[66] Once randomness is eliminated from the equation in this way, the return on the portfolio is equated to the return of a riskless asset based on the no-arbitrage assumption, and the equation is solved.[67] There are also binomial models for valuing a financial option that lead to equivalent solutions based on assumptions of no arbitrage.[68]

Pitkethly discusses the application of such methods for valuing financial options to real (that is, non-financial) options.[69] Pitkethly points out that, unlike the situation in the financial options case, there

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61. *See* Tonisson, *supra* note 54, at 20–21.

62. *See* PAUL WILMOTT ET AL., THE MATHEMATICS OF FINANCIAL DERIVATIVES 43 (1995).

63. *See id.* at 41–42, 48.

64. *Id.* at 78–80.

65. *Id*. at 79.

66. *Id*. at 42–43.

67. *Id*. at 42–43, 76–80.

68. *See* John C. Cox et al., *Option Pricing: A Simplified Approach*, 7 J. FIN. ECON. 229, 232–41, 246–54 (1979).

69. *See* Robert Pitkethly, *The Valuation of Patents: A Review of Patent Valuation Methods With Consideration of Option Based Methods and the Potential for Further Research* 1, 10–21 (Univ. of Cambridge Judge Inst. of Mgmt. Studies, Working Paper No. 21/97, 1997).

may not be a no-arbitrage equilibrium with a "spanning" portfolio of assets in the case of valuation of a patent, which might otherwise render the Black-Scholes analysis valid for valuing patents.[70] Additionally, Pitkethly indicates that the assumption of constant volatility of the underlying asset (that is, for call options, the volatility of the price of the corresponding share) may not be accurate in the context of patent valuation.[71] Further, Pitkethly points out that real options consist of multiple or compound options in a chain with numerous interdependencies.[72] Despite these difficulties and the complex nature of the problem, Pitkethly concludes that option-based valuation of patents should be possible.[73]

The literature includes attempts to use option theory to value patents in various contexts.[74] None of these appears to be fully satisfactory in providing an accurate value that would be accepted by all reasonable practitioners.[75]

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70. *See id.* at 14–15. The spanning portfolio of assets, in the case of financial options as discussed earlier, comprises a portfolio that includes the option and the underlying shares in a proportion that is chosen to eliminate randomness due to the lognormal random walk of the share's price. In the financial options case, this leads to a deterministic partial differential equation for the value of the option. *See id.*

71. *See id.* at 15–16.

72. For example, the post-grant phase of a patent may be treated as a series of options on the next years' benefits. Thus, patent value is considered in the context of a chain of multiple options. *Id.* at 19. Similarly, valuation of a patent application may be based on considering the consecutively arising options relating to paying the initial cost of the original application, the initial cost of foreign applications and the costs of preliminary examination and search, the cost of filing follow-on or continuation applications, and the cost of the grant fee. *Id*. at 20, 30.

73. *Id.* at 23–24.

74. *See, e.g.*, Denton, *supra* note 37, at 1179–80; *see generally* Alan C. Marco, *The Option Value of Patent Litigation: Theory and Evidence*, 14 REV. FIN. ECON. 323 (2005) (discussing a real options approach to patent litigation).

75. In Black-Scholes formalism, the value of the option is calculated as a function of the value of the present value of an underlying asset (*i.e.*, the stock). *See* F. Russel Denton & Paul J. Heald, *Random Walks, Non-Cooperative Games, and the Complex Mathematics of Patent Pricing*, 55 RUTGERS L. REV. 1175, 1194–95 (2003). The Denton article, in one section, argues that a patent is analogous to a stock option (and hence is not analogous to the underlying asset, which is the share). *Id*. However, in applying Black-Scholes formalism, the Denton article appears to treat the patent as the underlying asset, and the license of the patent as the option. *Id.* at 1203–19 (solving the Black-Scholes equation for C, the value of the option, which, in the Denton article's treatment, appears to be the price of a license to the patent). Further, the Denton article does not appear to justify in the patent context the no-arbitrage assumption for financial options, on which the Black-Scholes equation is based. *See supra* notes 62–69 for a discussion on the Black-Scholes equation and its derivation. The Marco article formulates and solves a mathematical model in which profit generated by the patent is assumed to follow geometric Brownian motion. *See* Alan C. Marco, *The Option Value of Patent Litigation: Theory and Evidence*, 14 REV. FIN. ECON. 323, 325 (2005). The model further assumes a terminal payoff value Ω that the patentee receives if the patentee asserts the patent in litigation, and that is discounted by the probability that the patent is invalid. *Id*. If, at any time, the present value of

III. THE PARCHOMOVSKY ARTICLE AND THE PORTFOLIO THEORY OF PATENT VALUATION

The Parchomovsky article notes that none of the known methodologies for valuing patents are satisfactory, and proposes an entirely different way of looking at the problem.[76] In particular, the Parchomovsky article proposes that "[t]he true value of patents inheres not in their individual worth, but in their aggregation into a collection of related patents—a patent portfolio."[77] By focusing on the portfolio, the Parchomovsky article was able to explain a number of trends in the patent system, including the patent paradox.[78]

A. *Patent Paradox*

The patent paradox is based on the observation that empirical research indicates that industry participants do not consider patents to be an effective protection mechanism, and that empirical studies suggest that the average value of a patent is quite small.[79] For example, one empirical work estimated that the mean patent value is around $4,000 for pharmaceutical patents, $5,000 for chemical patents, $15,000 for mechanical patents and $20,000 for electronics patents.[80] The Parchomovsky article points out that, nevertheless, the number of patent application filings in the United States and the world continues to increase, and that companies act as though patents are important.[81]

B. *Previous Attempts to Resolve the Patent Paradox*

Previous attempts to resolve the paradox do not seem to be satisfactory. For example, an explanation that patentees irrationally acquire patents despite their low value is likely not correct, given that the companies that participate in the most extensive patenting activity

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the patent drops below the terminal payoff value Ω, litigation is assumed to ensue. *Id*. at 325–26. This model leads to a stochastic differential equation that may be solved for the value of the patent. *Id*. at 327–29. The model is rigorous; however, the assumption that the value of the patent is zero, once asserted and once the terminal payoff is extracted appears to be unrealistic given that patents may be litigated multiple times, or licensed to non-litigants after assertion in litigation. *Id.* It is also not clear why profits derived from patents should follow geometric Brownian motion.

76. *See* Parchomovsky & Wagner, *supra* note 8, at 4–5.

77. *Id.* at 5–6.

78. *See generally id*.

79. *Id.* at 14.

80. *Id.* at 16-17 (citing Mark Schankerman, *How Valuable is Patent Protection? Estimates by Technology Field*, 29 RAND J. ECON. 77 (1998)).

81. *Id.* at 17–18.

are those that operate in the most competitive industries.[82] Needlessly wasting resources on a large scale on worthless patents would be expected to place such companies at a competitive disadvantage. However, no such competitive disadvantage is seen.[83]

In accordance with another theory, patents serve as a signaling device that the patentee is a high quality company, given the expense of obtaining patents.[84] Low quality companies would be unable to mimic such signals, given the expense of obtaining patents. Through such signals, the market is made to perceive value in the companies engaged in patenting. However, this rationale does not resolve the patent paradox, because it is not clear why the market would care about a company obtaining a commodity that does not have much intrinsic value.[85] In other words, obtaining and signaling the acquisition of patents should not act as a signal that the patenting company is valuable, given that patents on average are not valuable.[86]

According to another theory, the acquisition of patents serves the intra-company purpose of indicating to the company management that employees are doing their jobs.[87] Given that it is virtually impossible to directly measure employee effort, patents may be a useful quantifiable metric that the employees are performing. However, this rationale also does not resolve the patent paradox. If patents really are low value, as the evidence suggests, then equating patent application filings with successful job performance appears to be incorrect.[88] In fact, given the costs of patenting, employees who engage in patenting may be wasting valuable resources and should be directed to channel their energy for other company purposes.[89]

According to the lottery theory, patents are lottery tickets in that although most have negligible value, a few are so valuable that they provide a sufficient incentive for companies to undertake patenting activity.[90] The Parchomovsky article discusses a few different rationale for why that might not be the case, including the fact that most investors are risk averse and would not normally invest in activity that has a

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82. *Id.* at 19.

83. *Id*.

84. *Id*. at 20–22 (citing and discussing Clarissa Long, *Patent Signals*, 69 U. CHI. L. REV. 625 (2002)).

85. *Id.*

86. *Id*.

87. *Id*. at 22–24 (citing and discussing Richard C. Levin, *A New Look at the Patent System*, 76 AM. ECON. REV. (1986)).

88. *Id.*

89. *Id*.

90. *Id*. at 24–26 (citing and discussing F.M. Scherer, *The Innovation Lottery*, *in* EXPANDING THE BOUNDARIES OF INTELLECTUAL PROPERTY: INNOVATION POLICY FOR THE KNOWLEDGE SOCIETY 3 (Rochelle Cooper Dreyfuss et al. eds., 2001)).

negative expected value (unlike the typical lottery player).[91]

The Parchomovsky article also considers defensive patenting as an alternative theory for resolving the patent paradox. According to this theory, companies obtain patents to deter potential patent infringement suits. This theory provides considerable explanatory power; however, it only focuses on the defensive use of patents and fails to provide any rationale for obtaining a patent from an offensive perspective.[92] In particular, it does not provide any explanation of patent value other than the potential avoided costs patents might provide.

C. *Patent Portfolios and Their Benefits to Their Holders*

The Parchomovsky article divides the benefits of patent portfolios to their holders into two general classes: scale-related benefits and diversity-related benefits.[93] Perhaps most importantly as a scale-related benefit, a patent portfolio, especially when the various patent members are closely related, operates as a "super-patent" that excludes third parties from the collective scope of their claims.[94] The Parchomovsky article discusses other scale-related benefits, such as allowing a company to freely proceed along an innovation path in the cumulative technology area of the patent portfolio with less concern about infringing the patent rights of third parties; attracting innovations external to the company to the company through an easier process of licensing and acquisition; avoiding costly litigation because of the enhanced deterrence properties of a portfolio of patents compared to a single patent; improving bargaining position of the portfolio holder compared to competitors in the relevant market segment; attracting and retaining capital; and ensuring the company's voice is heard in relevant patent policy debates.[95]

The Parchomovsky article in particular points out that the litigation avoidance effect provided by a portfolio of patents relates to the fact that the win probability of the holder will increase with the increase in the portfolio size.[96] For example, assuming an average loss probability of 75% per asserted patent, a portfolio of just three patents would bring the likelihood of success of the portfolio holder above 50%.[97]

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91. Parchomovsky & Wagner, *supra* note 8, at 25–26.

92. *Id*.

93. *Id*. at 31–32.

94. *Id*. at 32–33.

95. *Id*. at 33–37.

96. *Id* at 35 n. 120.

97. This observation is a fundamental insight that motivates this Article. The Parchomovsky article considers this point in connection with the deterrence value of patents, but does not consider the calculus of probabilities of winning on the level of a discrete set of issues per patent in a litigation involving a portfolio of patents, and does not consider the impact of the

Regarding the diversity-related benefits of a portfolio of patents, the Parchomovsky article points out that the patents in a portfolio are not identical, and that the distribution of importance across the different patents that comprise the portfolio provides hedging against the risk and uncertainty that are bound together with innovation in the modern economy.[98] Specific diversity-related benefits include addressing ex ante uncertainty regarding whether specific technological features of a specific patent will turn out to be valuable;[99] expanding the freedom of research inquiry into a broader set of areas compared to that offered by a single patent; and addressing uncertainties related to future market conditions, future competitors and patent law.[100]

D. *Resolution of the Patent Paradox*

The Parchomovsky article's portfolio theory explains and resolves the patent paradox in an apparent way. Companies engage in patenting activity with great expenditure of resources, despite the low expected value of specific patents, because in so doing they effectively purchase the advantages of aggregation of the individual patents within the patent portfolio.[101] Companies acquiring patents and increasing the size of their portfolios are therefore being entirely rational, because in most cases, the marginal expected value of expanding the portfolio by one patent exceeds the marginal cost of obtaining that additional patent.[102]

E. *Predictions of the Portfolio Theory*

The Parchomovsky article, based on the portfolio theory of valuation, predicted that patent intensity will remain high, pressure on the PTO to increase patent quality will increase, patent thickets will continue to grow, patent litigation will become more complex and costly, mass licensing arrangements will proliferate, the patent system will increasingly favor large, well-funded, incumbent players, and the

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asymmetry in patent litigations undertaken by patent assertion entities. *See* discussion *infra* Part IV.

98. *See* Parchomovsky & Wagner, *supra* note 8, at 37–38.

99. *See id.* at 38. In particular, a portfolio of patents allows the patentee to address ex ante uncertainty about the features of a product that that will ultimately be successful. *Id*. For example, if many patent applications are filed directed to different features of a particular product, the patentee may be able to later in the process (*i.e.*, after filing an initial set of patents) file follow-on continuation applications to obtain comprehensive coverage of the features that are successful. Similarly, the patentee may be able to abandon the patents covering features of the product that are not successful.

100. *Id*. at 38–41.

101. *Id* at 52.

102. *Id*. at 42.

value of individual patents will become more obscure and increasingly irrelevant.[103]

Approximately 8 years after publication of the Parchomovsky article, many of these predictions are substantiated. For example, the total number of patent suits commenced has increased from around 3000 in 2005 to more than approximately 5000 in 2012.[104] Similarly, the number of patents granted has increased from around 170,000 to approximately 270,000 in 2012.[105] Although there appear to be no easily available, definitive statistics, patent litigation costs also appear to be escalating.[106] The quality of patents and the uncertainty of rights arising from low quality patents continue to be an issue.[107] Additionally, patent thickets are an increasingly articulated problem, and not just in the United States.[108] Most of the evidence available indicates that the predictions of the Parchomovsky article have been realized, and that they will continue to be relevant going forward.

IV. THE VIRTUAL CERTAINTY OF WINNING PORTFOLIO PATENT LITIGATION

The fundamental insight of the Parchomovsky article that motivates this Article is that the win probability in litigation of a holder of a portfolio of patents will increase with the increase in the portfolio size.[109] This Article considers this aspect in more detail below, and attempts to provide a more rigorous underlying quantitative basis. Furthermore, this Article posits that this principle, in combination with

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103. *Id* at 60–66.

104. *See* PRICEWATERHOUSECOOPERS, *supra* note 28, fig.1. *See also* EXECUTIVE OFFICE OF THE PRESIDENT, *supra* note 1, at 5 (providing statistics on the total number of patent cases filed and the percentage filed by PAEs).

105. *See* PRICEWATERHOUSECOOPERS, *supra* note 28, fig.1.

106. *See* Damon C. Andrews, *Why Patentees Litigate*, 12 COLUM. SCI. & TECH. L. REV. 219, 226–29 (2011); PRICEWATERHOUSECOOPERS, *supra* note 28, at 25. *See also* U.S. GOV'T ACCOUNTABILITY OFF., REPORT TO CONG. COMMS., INTELLECTUAL PROPERTY; ASSESSING FACTORS THAT AFFECT PATENT INFRINGEMENT LITIGATION COULD HELP IMPROVE PATENT QUALITY 25–26 (2011); EXECUTIVE OFFICE OF THE PRESIDENT, *supra* note 1, at 9.

107. *See* U.S. GOV'T ACCOUNTABILITY OFF., *supra* note 106, at 28–32, 39–45; EXECUTIVE OFFICE OF THE PRESIDENT, *supra* note 1, at 3.

108. *See, e.g.*, U.K. INTELL. PROP. OFF., A STUDY OF PATENT THICKETS (2013), *available at* http://www.ipo.gov.uk/ipresearch-thickets.pdf.

109. *See* Parchomovsky & Wagner, *supra* note 8, at 35 n.120. However, as discussed earlier, the Parchomovsky & Wagner article only considers this aspect in connection with the deterrence value of patents, and does not consider the calculus of probabilities of winning on the level of a discrete set of issues per patent in a litigation involving a portfolio of patents, and does not consider the impact of these result to the asymmetries of patent litigation undertaken by a PAE. *See supra* note 97.

the way the patent system currently works, virtually assures that PAEs will be successful in asserting a portfolio of patents against a targeted product or product line. A corollary of this is that a patent system in which an unlimited number of patents can be asserted against a single targeted product or product line will almost always result in patent liability. This result is tempered somewhat in the context of large-scale portfolio litigations of patentees against one another, because the end result usually approaches a stalemate that leads to, for example, a cross-licensing agreement. There is no such temperance in the context of PAE litigation, because of the asymmetry intrinsic to such litigation: given that the PAE does not make or sell products, the PAE itself cannot be subjected to a patent litigation suit. As a result, the PAE business model, assuming it is implemented properly, cannot fail—a PAE, by locating and acquiring a set of patents that has a nexus to the targeted product, should always succeed in patent litigation against the targeted product or product line. This may explain the proliferation of PAE litigation in recent years. The policy question this raises is whether such a patent system may be sustainably maintained.

A. *Basic Observations Relating to Current Patent Litigation*

One fundamental observation regarding patent litigation is that, by the time of trial, the dispute in litigation of a single patent is usually reduced to a small number of independent and material (that is, potentially outcome-determinative) issues—usually less than 20, and typically between 5-10—that must be resolved by the court during trial. Excepting actions for declaratory judgment in which an alleged infringer files the patent suit, the portfolio patentee usually has control over the number of these material issues and the probability of the patentee's succeeding on each of these material issues. This is because the patentee may engage in preparation and analysis before filing suit, and may select the set of patents that presents the highest probabilities for findings of infringement. In particular, the portfolio patentee may analyze a set of patents in the portfolio that have a nexus to the targeted, potentially infringing products, and pick one or more patents from that set that present the fewest number of material issues and the highest corresponding probabilities for prevailing on the material issues.[110]

Another basic observation is that patentees, especially PAEs, are

110. The patentee's ability to control the probabilities of winning the material issues that need to be resolved in the patent litigation is not critical to the analysis. In fact, as a matter of mathematical analysis, even a set of low probabilities for the patentee to prevail on the material issues in the litigation does not make a difference, as long as the patentee is able to find and assert a sufficiently large number of patents having a nexus to the targeted product or product line. *See* Tables 1 & 2, *infra* Part IV.

able to find and acquire the rights to more than one, and often, many patents that have a nexus to a targeted product or line of products. In particular, PAEs are often able to identify and assert a number of patents from different technology areas against the same product or product line.[111] This is especially true where the products involved are consumer goods in the hi-tech areas, such as cellular phones, televisions, printers, computers and laptops, which are complex products with multiple layers of components and functionalities that may incorporate a large number of patented technologies from different technology areas.[112]

When the patentee selects patents in different technology areas for assertion, it is likely that the material issues that arise in connection with a particular patent are independent from the material issues that arise in connection with the other patents. This is because the likelihood of correlation of material issues across patents should be low when the patents are not related to one another and relate to different technology areas.

Furthermore, by the time of trial, the material issues in dispute in connection with a specific patent can usually be reduced to a core set of independent material issues. For example, any claim construction disputes remaining by the time of trial will usually be independent from one another, because such disputes usually relate to distinct terms in the asserted claims of the patent. Similarly invalidity issues relating to different elements of a claim are also usually independent. It is frequently the case, however, that a material infringement or invalidity issue is based on a particular claim construction that is in dispute. In this case, the dispute regarding claim construction is not independent of the disputes regarding infringement or invalidity regarding the invalidity

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111. For example, in the International Trade Commission investigation *Certain Consumer Electronics and Display Devices and Products Containing Same*, the plaintiff PAE had asserted U.S. patent no. 6,650,327 directed to floating point rasterization in display image processing, U.S. patent no. 6,816,145 directed to LCD panel technology, and U.S. patent no. 5,717,881 directed to instruction processing within a CPU, against, among other things, various tablet devices. Certain Consumer Electronics Complaint, *supra* note 10. In the International Trade Commission investigation *Certain Electronic Devices, Including Handheld, Wireless Communications Devices*, the plaintiff PAE had asserted U.S. patent no. 5,235,635 directed to awakening a device from a low power state suing a keypad monitor with keypad activity-based activation, U.S. patent no. 5,530,597 directed to a technique for awakening a processor within an electronic device from an idle state and servicing high priority tasks using interrupts and masking techniques, and U.S. patent no. 5,608,873 directed to interprocessor communication against, among other things, certain wireless phones. Certain Electronic Devices, Including Handheld, Wireless Communications Devices, Inv. Nos. 337-TA-667 and 337-TA-673, *available at* http://www.itcblog.com/wp-content/uploads/2009/02/saxoncomplaint.pdf.

112. *See, e.g.*, Lemley, *supra* note 41, at 19–20; Mark A. Lemley & Carl Shapiro, *Patent Holdings and Royalty Stacking*, 85 TEX. L. REV. 1991, 2025–29 (2007).

argument.[113] However, this would not change the analysis below materially: the material issue regarding claim construction and the related material issue regarding infringement (or invalidity) would count as a single independent material issue instead of counting as two material issues. For that reason, the material issues in dispute in connection with an assertion of a specific patent can usually be reduced to a core set of independent material issues.

B. *Simple Illustrative Case—Two Material Issues Per Patent Asserted*

As a way of illustrating the basic idea, consider that the patentee is able to identify a plurality of patents that has a nexus to a particular product or line of products. Assume further that whether or not the product or product line infringes a particular asserted patent depends on the resolution of just two material issues.[114] These issues may comprise any of the typical material issues that are considered by courts during patent infringement trials, such as issues relating to claim construction, infringement, invalidity, and inequitable conduct. Assume that the probability of the patentee's prevailing on any of these issues, regardless of which patent is asserted, is given by p_ω, where $0< p_\omega <1$. It may be helpful, to illustrate the versatility of the analysis, to think of p_ω as being either a low probability such as 0.2 (that is, 20%), or a high probability such as 0.8 (that is, 80%). Consider the overall probability of the patentee's prevailing in obtaining a patent infringement judgment for at least one of the asserted patents, as the number of asserted patents is varied.

1. One Patent Asserted

When a single patent is asserted, the probability that the patentee loses in its infringement assertion is just the probability that the patentee loses on at least one of the two material issues (which is the same as one minus the probability that the patentee prevails on both material issues)[115]:

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113. *See* Roger A. Ford, *Patent Invalidity Versus Noninfringement*, 99 CORNELL L. REV. (forthcoming 2013), *available at* http://ssrn.com/abstract=2256207.

114. It is simplest, but not necessary, to consider that only a single claim is asserted per patent and that the material issues arising in connection with a patent all relate to the single asserted claim. The assertion of more than one claim per patent would lead to a greater likelihood of the patentee's success in proving patent infringement liability based on one claim of one patent. For that reason, there would be an even quicker convergence to 100% probability of the patentee's success in establishing liability of the accused infringer for at least one patent compared to the simple case in which a single claim is asserted per patent.

115. To establish infringement of a particular claim, the patentee needs to prevail on all the disputed material infringement issues relating to that claim. To establish that the claim is not

$$P_{lose} = (1 - p_{\omega}^{2})$$

So, the patentee's probability of winning a patent infringement judgment for the asserted patent is just one minus the above probability:

$$P_{win} = 1 - (1 - p_{\omega}^{2})$$

When the probability of patentee's winning each material issue is 80% (that is, $p_{\omega} = 0.8$), the patentee's probability of winning is 0.8 x 0.8 = 0.64, or 64%.

When the probability of patentee's winning each material issue is 20% (that is, $p_{\omega} = 0.2$), the patentee's probability of winning is 0.2 x 0.2 = 0.04, or 4%.

2. Two Patents Asserted

In this case two patents are asserted with each posing two material issues. The patentee's probability of losing its infringement case for both patents is:

$$P_{lose} = (1 - p_{\omega}^{2}) (1 - p_{\omega}^{2})$$

So, the patentee's probability of winning a patent infringement judgment for at least one of the patents is just one minus the above probability:

$$P_{win} = 1 - (1 - p_{\omega}^{2}) (1 - p_{\omega}^{2})$$

When the probability of patentee's prevailing on each material issue is 80% (that is, $p_{\omega} = 0.8$), the patentee's probability of winning on at least one patent is then 0.87, or 87%.

When the probability of patentee's prevailing on each material issue is 20% (that is, $p_{\omega} = 0.2$), the patentee's probability of winning is then 0.078, or 7.8%.

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invalid with respect to a particular reference, the patentee needs to prevail on only one of the disputed material invalidity issues. If there is more than one invalidity issue relating to a single asserted claim of a patent, then requiring that the patentee must win on all of the invalidity issues to prevail will lead to an underestimate of the patentee's chances of success. For that reason, the convergence to 100% probability that the patentee will succeed in establishing liability with respect to at least one patent will in actuality occur even more quickly than in the treatment that follows. For that reason, in the treatment that follows, it is permissible to consider all material invalidity issues that arise in connection with a single asserted claim as constituting a single, independent material issue that arises in connection with the claim.

3. Three Patents Asserted

In this case three patents are asserted, with each posing two material issues. The patentee's probability of losing its infringement case for all three patents is:

$$P_{lose} = (1 - p_{\omega}^2)\,(1 - p_{\omega}^2)\,(1 - p_{\omega}^2)$$

So, the patentee's probability of winning a patent infringement judgment for at least one of the patents is just one minus the above probability:

$$P_{win} = 1 - (1 - p_{\omega}^2)\,(1 - p_{\omega}^2)\,(1 - p_{\omega}^2)$$

When the probability of patentee's winning each material issue is 80% (that is, $p_{\omega} = 0.8$), the patentee's probability of winning on at least one patent is then 0.95, or 95%.

When the probability of patentee's winning each material issue is 20% (that is, $p_{\omega} = 0.2$), the patentee's probability of winning is then 0.115, or 11.5%.

4. N Patents Asserted

In this case N patents are asserted (where N is some positive integer number), with each posing two material issues. The patentee's probability of losing its infringement case for all N patents is[116]:

$$P_{lose} = (1 - p_{\omega}^2)\,(1 - p_{\omega}^2)\,\cdots\,(1 - p_{\omega}^2)$$

$$P_{lose} = \prod_{i=1}^{N} (1 - p_{\omega}^2)$$

So, the patentee's probability of winning a patent infringement judgment for at least one of the patents is just one minus the above probability:

$$P_{win} = 1 - \prod_{i=1}^{N} (1 - p_{\omega}^2)$$

Table 1 illustrates the probability of the patentee's winning on at least one patent when the probability of winning each material issue is 80% and 20%, respectively.

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116. This is $(1 - p_w^2)$ multiplied by itself N times.

Table 1 – Probability of Patentee's Winning on at Least one Patent When the Probability of Winning on Each of Two Issues is 20% or 80%

Number of Patents Asserted	Probability of Patentee's Winning on at Least One Patent when $p_\omega = 0.2$	Probability of Patentee's Winning on at Least One Patent when $p_\omega = 0.8$
1	4%	64%
2	7.8%	87%
3	11.5%	95.3%
4	15.1%	98.3%
5	18.5%	99.3%
6	21.7%	99.8%
7	24.9%	99.9%
8	27.9%	100%
9	30.7%	100%
10	33.5%	100%
20	55.8%	100%
30	70.6%	100%
40	80.5%	100%
50	87.0%	100%

As can be seen, in the model, when the probability of winning on each issue is $p_\omega = 80\%$, the patentee's probability of winning on at least one patent approaches 100% very quickly; in particular, when eight such patents are asserted, the patentee's probability of winning on at least one patent is approximately 100%.

When the probability of winning on each issue is $p_\omega = 20\%$, the patentee's probability of winning on at least one patent approaches 100% slowly; but when 10 such patents are asserted, the probability of the patentee's winning on at least one patent is approximately 33.5%, which is, after all, a large probability of winning considering the weak group of patents asserted.[117] It is obviously in the interest of the patentee to select for assertion the set of patents that provides the highest probabilities of winning on each of the relevant material issues.

117. Obviously, the expense and capability to sue on a large number of patents decrease as N increases. In particular, given the expense and complexity of U.S. patent litigation, it probably is not practically feasible to sue a defendant on more than about ten patents at a time.

Table 2 – Probability of Patentee's Winning on at Least one Patent When the Probability of Winning on Each of Five Issues is 20% or 80%

Number of Patents Asserted	Probability of Patentee's Winning on at Least One patent when $p_\omega = 0.2$	Probability of Patentee's Winning on at Least One Patent when $p_\omega = 0.8$
1	0.032%	32.8%
2	0.064%	54.8%
3	0.096%	69.6%
4	0.13%	79.6%
5	0.16%	86.3%
6	0.19%	90.8%
7	0.22%	93.8%
8	0.26%	95.8%
9	0.29%	97.2%
10	0.32%	98.1%
20	0.64%	100%
30	0.96%	100%
40	1.3%	100%
50	1.6%	100%
100	3.1%	100%
200	6.2%	100%
500	14.8%	100%
1000	27.4%	100%

C. *Dependence on the Number of Material Issues in Dispute*

As would be expected, an increase in the number of material issues in dispute leads to slower convergence of the probability of winning to 100% as the number of asserted patents becomes large. This can be seen by comparing Table 1 with Table 2, which treats the case of five material issues per patent, with the probability of winning on each

material issue being 20% or 80%.One can see that a greater number of issues in dispute per patent leads to slower convergence of the probability of the patentee's winning on at least one patent. The convergence, however, in all cases is still to a 100% probability. It is obviously in the accused infringer's interest to generate as many material issues in dispute as possible given the particular patents the PAE asserts in the litigation.

The convergence to 100% probability of the patentee's winning is extremely slow, when there are a large number of issues in dispute and the patentee's probability of winning each material issue is low. For example, as depicted in Table 2, in the case in which the patentee only has a 20% probability of winning on each of five material issues per patent, the patentee's probability of winning is only 1.6% when, unrealistically, 50 patents are asserted. Clearly, it is in the PAE's interest to take the time and care to diligently select the particular set of patents for assertion that maximizes the probabilities of the patentee's winning on all the material issues in dispute for that set of patents.

D. *Most General Case*

The above discussion treated a simple case in which a basic assumption is that there are exactly two or five issues in dispute per litigated patent. This is enough to illustrate the basic idea. However, these assumptions are not critical and only simplify the analysis. Appendix A of this Article treats the general case in which there are a potentially different number of material issues in dispute for each litigated patent, and there is a potentially different, independent probability of winning on each material issue. Under reasonable assumptions, it is easy to show that the probability of the patentee's winning on at least one patent approaches 100% in the limit that the number of patents becomes large. Furthermore, the sensitivity of the analysis to the probabilities on prevailing on the material issues approaches zero as the number of patents becomes large. For that reason, the exact probabilities on winning on each issue are not critical, as long as a sufficiently great number of patents are asserted.

V. THE NEAR CERTAINTY OF PAE VICTORY IN PORTFOLIO PATENT LITIGATION

The fundamental observation of this Article is that as the number of patents asserted against a product or line of products increases, the probability of the patentee's winning on at least one patent increases. In the limit of a large number of patents, this probability of winning

approaches 100%. Consequently, the patentee may ensure that it will win a patent infringement judgment in connection with a specific product or product line of an infringer by asserting a sufficiently large number of patents. Given that convergence to 100% probability of winning is slow when the patentee's probability of prevailing on material issues in dispute is low, the patentee should conduct due diligence to determine the group of patents that presents it with the highest probabilities of winning on the material issues in dispute.

The assumption of independence of probabilities of winning on material issues that underlies this result is likely most justified when the patentee asserts patents from unrelated technological areas against the same product or product line. This will most often be easiest to do when the product or product line involved is complex—for example, a hi-tech consumer good—such as a cellular phone, television, printer, computer or laptop. Such products include numerous electronic components, such as processors and storage components, software layers, displays, input/output devices, storage and memory components, each of which may be subject to multiple patents in different technology areas.[118]

A set of property rights that always or almost always results in liability for the accused infringer appears to be problematic. However, this result is tempered in most contexts by other considerations. First, there are many classes of products that do not have the multilayered complexity of hi-tech consumer goods. Because of this, the patent system does not tend to generate a large number of patents that cover such products. For example, pharmaceutical and mechanical products are not usually covered by a large number of patents. For that reason, the patentee in these contexts does not possess a strategy that is almost certain to lead it to prevail (or force a favorable settlement).

Second, in many contexts, the asymmetry in the probability of prevailing in a patent dispute that is discussed in this Article is alleviated by other considerations. One example is a dispute that involves the assertion of a portfolio of patents by the defendant in addition to the plaintiff, such as is the case when competitors sue each other for patent infringement. The fact that each party will likely prevail in its own assertion of its portfolio of patents will likely lead to a settlement, which is usually considered to be a useful social result.

There is one context where the asymmetry in the probability of

118. *See supra* note 111 and accompanying text. *See also* Jeffrey I.D. Lewis, *The Sky is not Falling: Navigating the Smartphone Patent Thicket*, WIPO MAG. (Feb. 2013), *available at* http://www.wipo.int/wipo_magazine/en/2013/01/article_0002.html (discussing the patent thicket in the smartphone product area); Carl Shapiro, *Navigating the Patent Thicket: Cross Licenses, Patent Pools, and Standard Setting*, *in* INNOVATION POLICY AND THE ECONOMY, 119, 119–22 (Adam B. Jaffe et al. eds. 2001) (discussing patent thickets and their consequences in various industries).

prevailing in a patent dispute is not alleviated by other considerations: portfolio patent litigation by a patent assertion entity (PAE) against an accused infringer that is a manufacturer and/or distributor of complex, multi-layered products. In this context, the PAE can assure itself of victory by asserting a plurality of patents against the accused infringer. The accused infringer, however, does not have any similar leverage against the PAE, because the PAE does not manufacture or distribute any products that are potentially vulnerable to a patent infringement suit.

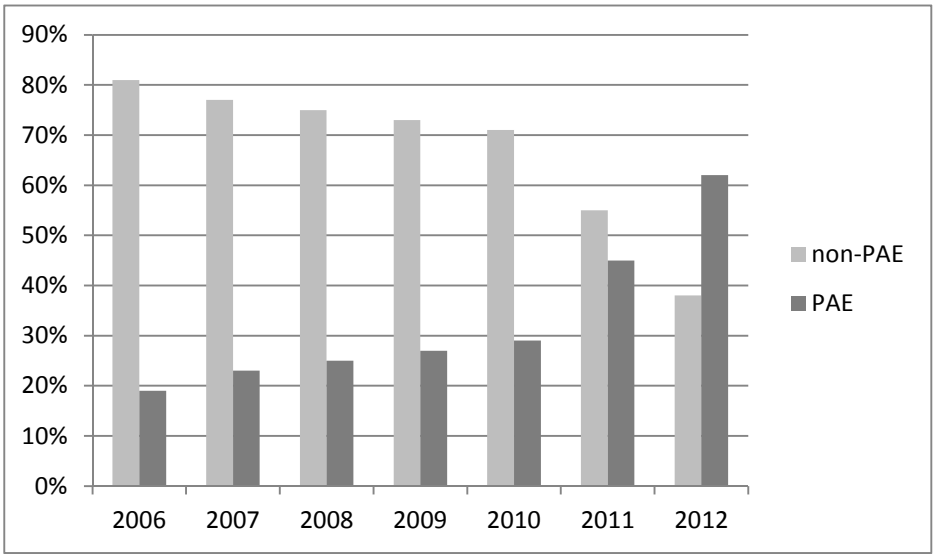

Fig. 1—Patent litigations filed by PAEs and non-PAEs by year (data from RPX Corp.)[119]

The prescription for the PAE is clear: assert a plurality of patents in different technology areas that all have a nexus to and potentially cover a commercially successful product or product line. This strategy should lead the PAE to either prevail in the litigation or force a favorable settlement. PAEs appear to frequently employ this strategy.[120] The tremendous increase in PAE lawsuits over the last few years may be a reflection of the reality that PAE victory is almost assured in portfolio patent litigation.[121]

119. *See* COLEEN V. CHIEN, PATENT TROLLS BY THE NUMBERS, SANTA CLARA UNIV. LEGAL STUDIES RESEARCH PAPER NO. 08-13, at 1 (2013), *available at* http://ssrn.com/abstract=2233041.

120. *See supra* note 111 and accompanying text.

121. *See* EXECUTIVE OFFICE OF THE PRESIDENT, *supra* note 1, at 5. The convergence to

VI. CONCLUSIONS

The primary conclusion of this Article is that PAE victory in portfolio patent litigation is virtually certain, as long as the PAE asserts a sufficient number of patents. This means that it will be in PAEs' interest to engage in portfolio patent litigation, as long as the potential damages exceed the costs of litigation. Given the prediction that the portfolio patentee will almost always prevail, this Article predicts that measures focused on penalizing a PAE that does not prevail, such as shifting the costs of litigation onto a losing portfolio patent-asserting PAE, will likely not succeed. Such measures may deter certain PAEs due to the perception of potential added risk that they entail; however, a determined PAE asserting sufficiently many patents should nevertheless prevail.

Furthermore, the theory discussed in this Article is consistent with many of the predictive insights of the Parchomovsky article. For example, as the Parchomovsky article predicted, patenting intensity has remained high during the years following the publication of that article in 2005. This Article predicts that this will continue to be the case going forward. Patenting intensity has been and is expected to continue to be especially high in technology areas where PAEs are most active, such as in the hi-tech and software technology areas.[122] Essentially, one would expect patentees to continue to apply for and obtain patents in technology areas where the patentee is virtually assured of victory in portfolio patent litigation.

Additionally, as again predicted by the Parchomovsky paper in 2005, pressure on the PTO regarding the quality and quantity of patents that are granted will continue to increase. This prediction of the Parchomovsky paper appears to have been fulfilled over the past eight years.[123] The theory discussed in this Article, however, implies that only

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100% probability of the patentee's establishing liability is slow when the patentee's probability of winning on the material issues presented by the asserted patents is low. However, it is likely, due to an increasing profusion of patent rights in relevant areas such as software and the hi-tech fields, that patentees are easily able to identify and assert patents that present material issues regarding which they have high probabilities of success.

Not all non-practicing entities are PAEs. In particular, universities and small inventors do not usually assert multiple patents against a product or a line of products like PAEs. However, only 5% or so of the cases filed by non-practicing entities during 1995-2012 were filed by universities. *See* PRICEWATERHOUSECOOPERS, *supra* note 28, at 27.

122. *See* JAMES BESSEN, A GENERATION OF SOFTWARE PATENTS, BOSTON UNIVERSITY SCHOOL OF LAW WORKING PAPER NO. 11-31, at 26 (2011), *available at* http://www.bu.edu/law/faculty/scholarship/workingpapers/documents/BessenJ062111withBeckman.pdf (indicating that approximately 2000 software patents were granted in the United States in 1980, approximately 25,000 were granted in 2005, and approximately 37,000 were granted in 2009).

123. *See, e.g.*, U.S. GOV'T ACCOUNTABILITY OFF., *supra* note 106, at 39–45 (discussing measures that the PTO has taken or is undertaking to improve patent quality).

drastic reform in the area of the quality and quantity of patents granted by the PTO can potentially make a difference. For example, virtual certainty of victory of the portfolio patent holder in patent litigation can only be prevented if no more than a few patents are assertable against complex products, especially in the hi-tech and software areas. No patent reform effort discussed to this point, short of a previously discussed, radical and problematic outright ban on patents in these areas would likely make a material difference in this regard.[124] In particular, changes in the obviousness threshold for patentability are unlikely to make much of a difference, unless such changes result in only a few granted patents being assertable against complex products in the hi-tech and software areas.[125]

Measures that prevent PAEs from asserting patents, or prevent them from asserting more than one or a few patents against a single product or product line, could potentially alleviate the problem discussed in this Article. However, it may be difficult to provide for a clear set of rules that has the desired effect. For example, a PAE may assert a portfolio of patents against a product or product line in separate litigations such that only a single patent is asserted in each of the litigations. Furthermore, a PAE may collude with non-PAEs in asserting a portfolio of patents against a product or product line. In particular, a PAE could assign patents it believes have a nexus to a particular product or product line to a non-PAE entity in return for promises by the latter to assert the patents and share any damages revenues that the latter obtains.

Remedies-oriented measures may also disincentivize PAEs from asserting patents, including portfolios of patents. For example, a lower measure of damages, and/or rendering it more difficult for PAEs to obtain injunctions, may change the cost/benefit calculus for PAEs in a way that discourages the filing of patent suits, including those involving a portfolio of patents. This Article predicts, however, that as long as the potential damages that could be collected continues to materially exceed the total costs of portfolio patent litigation, PAEs will continue to assert portfolios of patents against particular products or product lines.

Finally, a corollary of the result that a PAE is assured of victory by

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124. Certain authors argue that the threshold for patentability of software patents should be set higher, without arguing for a complete ban of software patents. *See, e.g.*, BEN KLEMENS, THE RISE OF THE INFORMATION PROCESSING PATENT 38, (June 21, 2007), *available at* http://ssrn.com/abstract=959931 (arguing that a device should be patentable "only if it is based on steps that are simultaneously novel and non-trivially physical"). An outright ban could have an adverse impact on innovation, because, for example, it could make it more difficult for startup companies engaged in software innovation or using software to obtain financing.

125. The U.S. Supreme Court raised the obviousness threshold for patentability materially in 2007. KSR Int'l Co. v. Teleflex, Inc., 550 U.S. 398 (2007). However, while approximately 30,000 software patents were granted in 2006, approximately 37,000 were granted in 2009. BESSEN, *supra* note 123, at 26.

asserting a sufficient number of patents is that the value of a specific patent likely has strong dependence on how it is asserted. For example, a patent that does not have great value based on an analysis using conventional valuation techniques may actually help establish liability and large patent damages when it is asserted by a PAE together with other patents. For that reason, known methodologies for valuing patents, to the extent they do not consider how the value proposition may be impacted by assertion of the patent together with other patents, may be deficient.

Appendix A

Mathematical Treatment of the General Case

The patentee asserts N patents, where N is some positive integer number. The determination of whether the i^{th} patent is infringed requires the resolution of n_i material legal issues in dispute, where each n_i is a positive integer number. These issues may comprise any of the typical material issues that are considered by courts during patent infringement trials, such as claim construction, infringement, invalidity, inequitable conduct and so on.[126] The probability of the patentee's winning on the j^{th} legal issue for the i^{th} patent is denoted by p_j^i.

The N patents are from different technology areas, so that the probabilities for prevailing on the material legal issues of different patents are most likely mathematically independent of one another. Further, the material legal issues associated with a single patent from among the N patents are also assumed to be mathematically independent.

The probability of the patentee losing on all material issues (and thus on all patents) is then:

$$P_{lose} = \prod_{i=1}^{N}\left(1 - \prod_{j=1}^{n_i} p_j^i\right)$$

Assume that $0 < p_j^i < 1$. This means that there is some uncertainty in winning or losing on each of the material legal issues. Then, for all i,

$$0 < \left(1 - \prod_{j=1}^{n_i} p_j^i\right) < 1$$

126. As discussed in the main body of this paper, a patent infringement dispute may reduce to a core set of independent material issues before trial. *See supra* text accompanying note 113. In connection with a particular claim, the patentee must generally win on all of the material infringement-related issues to establish liability. However, if there is more than one invalidity issue relating to a single asserted claim of a patent, then requiring that the patentee must win on all of the invalidity issues to prevail will lead to an underestimate of the patentee's chances of success. For that reason, the convergence to 100% probability that the patentee will succeed in establishing liability with respect to at least one patent will in actuality occur even more quickly than in the treatment that follows. For that reason, in the treatment that follows, it is permissible to consider all material invalidity issues that arise in connection with a single asserted claim as constituting a single, independent material issue that arises in connection with the claim. *See supra* note 115.

In particular, in realistic scenarios, there will be some particular value p_{max} such that for each i:

$$0 < \left(1 - \prod_{j=1}^{n_i} p_j^i\right) \leq p_{max} < 1$$

Then,

$$\lim_{N\to\infty} \prod_{i=1}^{N} \left(1 - \prod_{j=1}^{n_i} p_j^i\right) \leq \lim_{N\to\infty} \prod_{i=1}^{N} p_{max} = \lim_{N\to\infty} {p_{max}}^N$$

But, $\lim_{N\to\infty} {p_{max}}^N = 0$.[127] So,

$$\lim_{N\to\infty} P_{lose} = \lim_{N\to\infty} \prod_{i=1}^{N} \left(1 - \prod_{j=1}^{n_i} p_j^i\right) = 0$$

Thus, the probability of the patentee's losing on all N patents approaches zero. This is the just a different way of stating that the probability of the patentee's winning on at least one patent is 100% in the limit that the number of patents asserted approaches infinity.

Sensitivity of the Conclusion to Errors in Assessing the Probability of the Patentee's Winning on a Material Issue

This section of the paper considers the sensitivity of the conclusion that a PAE is assured of victory to any errors in assessing the probabilities of the patentee's winning on the material issues that arise in connection with the assertion of each patent. As discussed below, for a large number of patents that are asserted, the sensitivity to the exact values of these probabilities approaches zero. Thus, the greater the number of patents that are asserted, the lesser the importance of determining the exact values of these probabilities.[128]

A. The Case of Two Material Issues Per Patent and Equal Probabilities

As discussed in the main body of this paper in Part IV, when there are two material issues per patent and the probability of the patentee's

127. In particular, for any $\varepsilon > 0$, choose any integer $m > \frac{\ln \varepsilon}{\ln p_{max}}$. Then, ${p_{max}}^m < \varepsilon$. Thus, ${p_{max}}^m$ can be made arbitrarily close to 0 by choosing m sufficiently large.

128. Given that the functions P_{lose} considered in this paper are well-behaved, and that, as discussed earlier, $\lim_{N\to\infty} P_{lose} = 0$ it is not surprising that the sensitivity of P_{lose} to changes in the value of p_ω goes to zero as $N \to \infty$. The following provides a specific treatment.

prevailing on each issue is given by p_ω, the patentee's probability of losing on all asserted patents is given by:

$$\text{Probability of patentee's losing} = P_{lose} = \prod_{i=1}^{N}(1 - {p_\omega}^2)$$

Then, to first order, the sensitivity of P_{lose} to the value of p_ω is given by:

$$\frac{\partial P_{lose}}{\partial p_\omega} = \frac{-2Np_\omega}{(1 - {p_\omega}^2)^{1-N}}$$

In the limit as $N \to \infty$ both the numerator and the denominator go to infinity. For that reason, one may apply L'Hôpital's Rule, which involves considering the limit when $N \to \infty$ of the fraction formed by the derivative with respect to N of the numerator and the derivative with respect to N of the denominator:

$$\lim_{N\to\infty} \frac{\partial P_{lose}}{\partial p_\omega} = \lim_{N\to\infty} \frac{2p_\omega}{\ln(1 - {p_\omega}^2)(1 - {p_\omega}^2)^{1-N}} = 0$$

Thus, there is no sensitivity to first order of the patentee's probability of losing (and thus also of the patentee's probability of winning on at least one patent) to errors in the exact value of the probability p_ω in the limit of a large number of asserted patents.

Higher Order Derivatives

It can be shown that the higher-order derivatives of P_{lose} with respect to p_ω also approach zero as N approaches infinity. For example, it can be shown that:

$$\frac{\partial^2 P_{lose}}{\partial {p_\omega}^2} = \frac{-2N}{(1 - {p_\omega}^2)^{1-N}} + \frac{4(N)(N-1){p_\omega}^2}{(1 - {p_\omega}^2)^{2-N}}$$

$$\frac{\partial^3 P_{lose}}{\partial {p_\omega}^3} = \frac{12p_\omega(N)(N-1)}{(1 - {p_\omega}^2)^{2-N}} - \frac{8(N)(N-1)(N-2){p_\omega}^3}{(1 - {p_\omega}^2)^{3-N}}$$

$$\frac{\partial^4 P_{lose}}{\partial {p_\omega}^4} = \frac{12(N)(N-1)}{(1 - {p_\omega}^2)^{2-N}} - \frac{48(N)(N-1)(N-2){p_\omega}^2}{(1 - {p_\omega}^2)^{3-N}} + \frac{16(N)(N-1)(N-2)(N-3){p_\omega}^4}{(1 - {p_\omega}^2)^{4-N}}$$

Based on the above, it is apparent that the highest order term of the n^{th} order derivative will be of order $(1 - {p_\omega}^2)^N \cdot N^n$. Repeated application of L'Hôpital's Rule indicates that this term will go to zero as N

approaches infinity. For that reason, higher order derivatives also approach zero as N approaches infinity.

B. The Case of M Material Issues Per Patent and Equal Probabilities

The analysis in this case is substantially similar to that of the case of two material issues per patent. The patentee's probability of losing on all asserted patents is given by:

$$\text{Probability of patentee's losing} = P_{lose} = \prod_{i=1}^{N}(1 - {p_\omega}^M)$$

Then, to first order, the sensitivity of the patentee's probability of losing on all N patents to the value p_ω is given by:

$$\frac{\partial P_{lose}}{\partial p_\omega} = \frac{-NM{p_\omega}^{M-1}}{(1 - {p_\omega}^M)^{1-N}}$$

In the limit as $N \to \infty$ both the numerator and the denominator go to infinity. For that reason, once again apply L'Hôpital's Rule:

$$\lim_{N\to\infty} \frac{\partial P_{lose}}{\partial p_\omega} = \lim_{N\to\infty} \frac{M{p_\omega}^{M-1}}{\ln(1 - {p_\omega}^M)(1 - {p_\omega}^M)^{1-N}} = 0$$

Thus, in this case as well, there is no first order sensitivity of the patentee's probability of losing (and thus also of the patentee's probability of winning on at least one patent) to errors in the exact value of the probability p_ω in the limit of a large number of asserted patents.

Higher Order Derivatives

It can be shown that the higher-order derivatives of P_{lose} with respect to p_ω also approach zero as N approaches infinity. For example, it can be shown that:

$$\frac{\partial^2 P_{lose}}{\partial {p_\omega}^2} = \frac{-N(M)(M-1){p_\omega}^{M-2}}{(1 - {p_\omega}^M)^{1-N}} + \frac{M^2(N)(N-1){p_\omega}^{2M-2}}{(1 - {p_\omega}^M)^{2-N}}$$

$$\frac{\partial^3 P_{lose}}{\partial {p_\omega}^3} = \frac{-N(M)(M-1)(M-2){p_\omega}^{M-3}}{(1 - {p_\omega}^M)^{1-N}} + \frac{3(N)(N-1)(M^2)(M-1){p_\omega}^{2M-3}}{(1 - {p_\omega}^M)^{2-N}} - \frac{(N)(N-1)(N-2)M^3{p_\omega}^{3M-3}}{(1 - {p_\omega}^M)^{3-N}}$$

Based on the above, it is apparent that the highest order term of the nth order derivative will be of order $(1 - {p_\omega}^M)^N \cdot N^n$. Repeated application of L'Hôpital's Rule indicates that this term will go to zero as N approaches infinity. For that reason, higher order derivatives also approach zero as N approaches infinity.

C. The Case of an Arbitrary Number of Material Issues Per Patent and Equal Probabilities of Prevailing on the Material Issues in Dispute

In this case, the ith patent presents a_i material issues that are disputed by the patentee and the accused infringer. The probability of the patentee's prevailing on each material issue is given by p_ω. Then,

$$P_{lose} = \prod_{i=1}^{N} (1 - {p_\omega}^{a_i})$$

Note that for each patent, there will be a finite number of issues in dispute. For that reason, one can rewrite the terms of the expression above so that the factors representing one issue per patent appear first, followed by factors representing two issues per patent, three issues per patent and so on:

$$P_{lose} = (1 - {p_\omega}^1)^{b_1}(1 - {p_\omega}^2)^{b_2}(1 - {p_\omega}^3)^{b_3} \cdots (1 - {p_\omega}^{max})^{b_{max}}$$

This expression describes the fact that there are b_1 patents that have only a single material issue in dispute, b_2 patents that have two material issues in dispute and so on. Accordingly, there are b_{max} patents that have max number of material issues in dispute, which is the maximum number of material issues in dispute that any one patent from the set of N patents may have. Clearly,

$$\sum_{i=1}^{max} i \cdot b_i = \sum_{j=1}^{N} a_j$$

Then, to first order, the sensitivity of the patentee's probability of losing on all N patents to the value of the probability of winning on a material issue is given by:

$$\frac{\partial P_{lose}}{\partial p_\omega} = -\frac{b_1}{1-{p_\omega}^1}P_{lose} - \frac{2p_\omega b_2}{1-{p_\omega}^2}P_{lose} - \frac{3{p_\omega}^2 b_3}{1-{p_\omega}^3}P_{lose} \cdots - \frac{max \cdot {p_\omega}^{max-1} b_{max}}{1-{p_\omega}^{max}}P_{lose}$$

Simplifying,

$$\frac{\partial P_{lose}}{\partial p_\omega} = -P_{lose} \cdot \sum_{i=1}^{max} \frac{i \cdot p_w^{i-1} \cdot b_i}{1-{p_\omega}^i}$$

Because $0 < {p_\omega}^i < 1$,

$$\left|\frac{\partial P_{lose}}{\partial p_\omega}\right| = P_{lose} \cdot \sum_{i=1}^{max} \frac{i \cdot p_w^{i-1} \cdot b_i}{1-{p_\omega}^i} \leq P_{lose} \cdot \sum_{i=1}^{max} \frac{i \cdot p_w^{i-1} \cdot b_i}{1-{p_\omega}^1} = \frac{P_{lose}}{1-{p_\omega}^1} \sum_{i=1}^{max} i \cdot p_w^{i-1} \cdot b_i$$

It is easy to show that the function $f(i) = i \cdot p_w^{i-1}$ reaches a maximum value at the nearest integer value of i that is greater than $-\frac{1}{\ln p_\omega}$. Calling this nearest integer value i_0,

$$\left|\frac{\partial P_{lose}}{\partial p_\omega}\right| \leq \frac{P_{lose}}{1-{p_\omega}^1} \sum_{i=1}^{max} i \cdot p_w^{i-1} \cdot b_i \leq \frac{P_{lose} \cdot i_0 \cdot p_w^{i_0-1}}{1-{p_\omega}^1} \sum_{i=1}^{max} b_i$$

But,

$$\sum_{i=1}^{max} b_i = N$$

So, for some finite constant value C,

$$\left|\frac{\partial P_{lose}}{\partial p_\omega}\right| \leq C \cdot P_{lose} \cdot N$$

But,

$$P_{lose} = \prod_{i=1}^{N}(1-{p_\omega}^{a_i}) \leq \prod_{i=1}^{N}(1-{p_\omega}^{b_{max}}) = (1-{p_\omega}^{b_{max}})^N$$

So,

$$\lim_{N\to\infty}\left|\frac{\partial P_{lose}}{\partial p_\omega}\right| \le C \cdot \lim_{N\to\infty}(P_{lose}\cdot N) \le C \cdot \lim_{N\to\infty}[(1-{p_\omega}^{b_{max}})^N \cdot N]$$

It is easy to see based on the above, by repeatedly applying L'Hôpital's Rule, that

$$\lim_{N\to\infty}\left|\frac{\partial P_{lose}}{\partial p_\omega}\right| = 0$$

Therefore, in this case as well, there is no sensitivity to first order of the patentee's probability of losing (and thus also of the patentee's probability of winning on at least one patent) to errors in the exact value of the probability p_ω in the limit of a large number of asserted patents.

Higher-Order Derivatives

It can be shown that the higher-order derivatives of P_{lose} with respect to p_ω also approach zero as N approaches infinity. For example,

$$\frac{\partial^2 P_{lose}}{\partial {p_\omega}^2} = -\frac{\partial P_{lose}}{\partial p_\omega}\sum_{i=1}^{max}\frac{i\cdot p_w^{i-1}\cdot b_i}{1-{p_\omega}^i} - P_{lose}\sum_{i=1}^{max}\frac{i^2\cdot p_w^{i-2}\cdot b_i - i\cdot p_w^{i-2}\cdot b_i + i\cdot p_w^{2i-2}\cdot b_i}{1-{p_\omega}^i}$$

The first term on the right hand side is of order $P_{lose}\cdot N^2$. By once again applying L'Hôpital's Rule repeatedly, one can show that the limit as N approaches infinity of the first term on the right hand side is zero. Additionally, the limit as N approaches infinity of the second term on the right hand side is also zero. Given that max, the maximum number of issues in dispute that any one patent from the set of patents may have, is finite, the second term on the right hand side is of order $P_{lose}\cdot N$. As discussed earlier, the limit as N approaches infinity of $P_{lose}\cdot N$ is zero (based on an application of L'Hôpital's Rule). For that reason, the limit as N approaches infinity of the second term on the right hand side is also zero.
Thus,

$$\lim_{N\to\infty}\left|\frac{\partial^2 P_{lose}}{\partial {p_\omega}^2}\right| = 0$$

Additionally, it is apparent, based on the expression for $\frac{\partial^2 P_{lose}}{\partial {p_\omega}^2}$ above,

that the highest order term of the n^{th} order derivative will be of order $P_{lose} \cdot N^n$. Repeated application of L'Hôpital's Rule indicates that this term will go to zero as N approaches infinity. For that reason, higher order derivatives also approach zero as N approaches infinity.

D. The Case of an Arbitrary Number of Material Issues Per Patent With Respective Probabilities of Prevailing on the Material Issues in Dispute

As was discussed earlier in this Appendix, in the most general case, the determination of whether the i^{th} patent is infringed requires the resolution of n_i material legal issues in dispute, where each n_i is a positive integer number. The probability of the patentee's winning on the j^{th} legal issue for the i^{th} patent is denoted by p_j^i.

The probability of the patentee losing on all issues (and thus on all patents) is then:

$$P_{lose} = \prod_{i=1}^{N} \left(1 - \prod_{j=1}^{n_i} p_j^i \right)$$

Then, to first order, the sensitivity of the patentee's probability of losing on all N patents to the value of the probability of winning on a particular material issue is given by:

$$\frac{\partial P_{lose}}{\partial p_{j_0}^{i_0}} = P_{lose} \cdot \frac{-\prod_{j \neq j_0} p_j^{i_0}}{1 - \prod_j p_j^{i_0}}$$

Clearly, for some constant K,

$$\lim_{N \to \infty} \left| \frac{\partial P_{lose}}{\partial p_{j_0}^{i_0}} \right| = K \cdot \lim_{N \to \infty} P_{lose}$$

But, as discussed earlier at the beginning of this Appendix (in connection with the mathematical treatment of the general case), for this case,

$$\lim_{N \to \infty} P_{lose} = 0$$

Therefore,

$$\lim_{N \to \infty} \left| \frac{\partial P_{lose}}{\partial p_{j_0}^{i_0}} \right| = 0$$

Thus, in this case as well, there is no first-order sensitivity of the patentee's probability of losing (and thus also of the patentee's probability of winning on at least one patent) to any error in the exact value of the probability p_j^i, in the limit of a large number of asserted patents.

Higher-Order Derivatives

It can be shown that the higher-order derivatives of P_{lose} with respect to the probability p_j^i also approach zero as N approaches infinity. First, because the expression for P_{lose} is linear with respect to any particular p_j^i, any second or higher order derivative with respect to a particular $p_j^i$is zero. Second, it is easy to see that any mixed partial derivative of P_{lose} that includes first order derivatives with respect to different p_j^i s will be at most of order P_{lose}. For that reason, higher-order derivatives of P_{lose} with respect to the probabilities p_j^i also approach zero as N approaches infinity.